OptionMetrics

Dividend Forecasts, Option Pricing Models, And Implied Volatility Calculations:

Why simpler is better

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Introduction

Option models and implied volatility calculations have been around for more than a quarter of a century. The Black-Scholes pricing model, which started the whole modern enterprise, was predicated on the assumption that the underlying security paid no dividends, and that the risk-free rate of interest faced by option holders was constant and identical for all maturities.

Despite these somewhat unrealistic assumptions, the Black-Scholes model for European options, along with its American option counterpart, the Cox-Ross-Rubinstein binomial tree, is still considered by most option market participants to be the technique of choice for determining the implied volatility of a traded equity or index option. The original approach has been modified to allow for a non-flat yield curve and the incorporation of discrete dividend payments by the underlying security. In the case of interest rates, most market participants are guided by the observations of Black, who argued that the constant risk-free interest rate required by the calculation should be set equal to the interest rate that correctly prices zero-coupon bonds that mature at the same time the option expires.

In the case of discrete dividend payments to holders of the underlying security, there is much more diversity of opinion. While it is generally agreed that the actual timing of the payments is important, and should be incorporated into the option pricing model, there is disagreement about the correct way to model the size of a future dividend payment when the actual dividend is expected but has not yet been declared by the firm.

Many of the larger investment banks and institutions have expended a great deal of time and effort on sophisticated dividend forecasting models and methodologies. These models are often used in equity valuation, since a large family of equity valuation models (based on the Gordon growth model and its variants) is based on the rate of growth of future dividends. While forecasting the timing of the dividend payments is relatively straightforward (since most firms pay dividends according to a regular schedule), the sophistication of these proprietary models is in their ability to correctly anticipate some of the nuances of each firm's dividend policy, such as the tendency for corporate boards to increase dividend only once per year, the desire by boards to keep dividend payout ratios close to a predetermined target, and their general reluctance to reduce dividends in the face of temporary adverse business conditions.

Given that such dividend forecasting models exist, is their use appropriate for the pricing of options? In this paper, we argue not only that these models are more sophisticated then necessary for the task at hand, but also that they can lead to option prices and implied volatilities which are inconsistent or incorrect. Instead, we propose a simpler approach, which has been presented previously in the option pricing literature, namely that of assuming that the dividend yield on the underlying security remains constant. We argue that this approach for discrete dividend payments is more consistent with the other Black-Scholes modeling assumptions.

Discrete dividends and option models

While there no doubt a number of approaches for handling discrete dividend payments in option models, here we present and discuss two of the more popular ones: fixed dividend payments, and constant dividend yields.

Fixed dividend payments

In this approach, the future dividends are assumed to be fixed, deterministic, and unrelated to any future underlying price movements. The future dividends may be constant, or more typically they may reflect a growth rate derived from a dividend forecasting model.

A major advantage to this approach is that it is relatively easily incorporated into the Cox-Ross-Rubinstein binomial tree model using an approach suggested by Black. At each node of the binomial tree, the value of the underlying security price at that node is adjusted by subtracting the present value of all future dividends from that point forward, discounted by the risk-free rate. Using these adjusted prices, the option is valued as usual.

As simple as this approach seems, a critical piece of information is missing: how are future dividend payments to be determined? At very least, we would need some sort of dividend forecasting model, even if a rudimentary one. Even if dividends could be forecast with perfect accuracy, it is unclear that the actual future dividend growth rate is the appropriate growth rate to use in a single-factor risk-neutral model of option prices. Since the underlying stock price is at least partially determined by future dividend payout rates, the growth in dividends should somehow be tied to the average growth in the underlying stock price. But in a risk-neutral model, the underlying stock price grows at the risk-free rate; and consequently, so should the dividends.

Even if a correct risk-neutral dividend growth rate is selected, can we really treat the dividends as deterministic? It does not seem at all probable that a dividend payment issued by a corporate board one year from today would be completely independent of the stock price in one year. At very least, the stochastic nature of stock price movements inherent in the model should carry over to the dividends as well. These considerations would argue at very least for a dividend forecast which is dependent on the price of the stock at the time of the payout, and then we would still have the issue of the appropriate risk-neutral growth rate.

A more sophisticated approach to dividend modeling might incorporate path-dependence, so that the dividend payout is based on the current payout ratio as well as previous payouts and the stock price history. Or, one might include a second source of risk and treat the dividend as a stochastic variable. While both approaches are worthy of consideration, they require significant changes to the underlying option pricing model which would take us far beyond the original Black-Scholes methodology. Indeed, such models are more akin to the stochastic volatility models that have been introduced in the literature but as of yet have not been widely accepted by market participants.

Constant dividend yields

Under the most common alternative approach, one assumes that the current dividend yield remains constant over the future life of the option. This approach captures the intuition the growth rate in the dividend is closely tied to the growth rate of the stock. In particular, the Gordon growth model predicts that for firms in steady-state growth, a constant dividend yield is consistent with a constant average return on equity and a constant dividend growth rate. Since dividends are often paid by mature firms that exhibit relatively stable growth, these predictions are consistent with our intuitions about stock prices. The justification for this approach also follows from our empirical observations that stock prices are tied closely to earnings, and that corporate boards tend to attempt to keep payout ratios tied to a constant target.

By tying the value of the dividend payment directly to the value of the stock via a constant dividend yield, we can ignore the issues of the appropriate growth rate. Because the dividend payment is a constant proportion of the stock price, it grows at the risk-free rate in a risk-neutral world. In the same sense that the actual growth rate of the stock is unimportant to the pricing of the option (according to Black and Scholes celebrated analysis), the growth rate of the dividend is also unimportant under the same fundamental assumptions. This risk-neutral growth assumption means that the dividend payments can easily be incorporated into the option model with no modifications.

Implied Volatility and Dividend Estimation

How close do we need to be in estimating dividends? The answer depends on the nature of the problem we are trying to solve. For option price modeling, the sensitivity of the theoretical option price with respect to the dividend yield is quite small (except for very long-dated options), so we need not be too concerned if our estimate is inaccurate, even by as much as 50 basis points or more. However, the question needs to be more closely examined when implied volatility calculation is the goal. Clearly, we need to be much more concerned about estimation error in dividend modeling if a small error in dividend forecasts leads to a large error in implied volatility.

To help answer this question, we will assume that we have a European option on a stock for which the original Black-Scholes conditions hold, and which pays continuous dividend at a constant rate. These assumptions make the analysis more tractable, but the results extend directly to the case of an American option and discrete dividend payments.

Using the Black-Scholes formula, it is straightforward to show that for an at-the-money forward option (delta = 0.5) with an expiration of T years, the sensitivity of the implied volatility is given by

$$\frac{\partial \sigma}{\partial q} = \frac{\partial C}{\partial q} \Big/ \frac{\partial C}{\partial \sigma} \approx -1.253\sqrt{T}e^{-qT}$$

Thus, for a one-year option with an estimated dividend yield of 2.0%, an error in the estimated dividend rate equal to 25 basis points (0.25%) would lead to an error in the implied volatility of approximately 0.32% (0.32 volatility points). Clearly, we have quite a bit of leeway in our estimation of dividend yields.

What about the error for an in-the-money option? In this case, the implied volatility can be extremely sensitive to the input parameters due to the very low vega/kappa, or volatility sensitivity, of the option price. By explicitly computing vega and rho, we find that a one-year in-the-money (delta = 0.75) option would have an error of only 0.59% (0.59 volatility points) for a 25 basis point error in dividend yield. While the in-the-money option is more sensitive to dividend yield estimation error, the overall sensitivity is still quite small.

Conclusion

For the purposes of calculating option prices or implied volatilities, the use of a dividend forecasting model based on projected actual dividend growth rates can lead to an option model which is internally inconsistent. In contrast, the use of a model based on constant dividend yields is not only consistent, but also easier to implement. Regardless of the method of handling future dividend payments, any associated errors in the forecast dividend rate will in general result have only a small impact on calculated option implied volatilities, even for long-dated options.